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Enumeration of large mixed four-and-two-level regular designs

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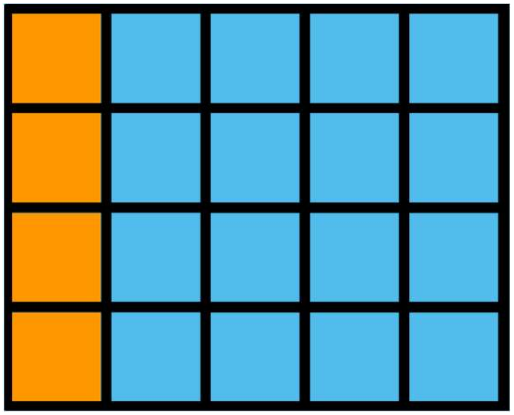
Outline

- Motivation
- Methodology
- Selection procedure
- Catalog
- Conclusion

Four-and-two-level designs

A four-and-two-level design $4^m 2^n$ has:

- m four-level factors
- n two-level factors
- Regular designs



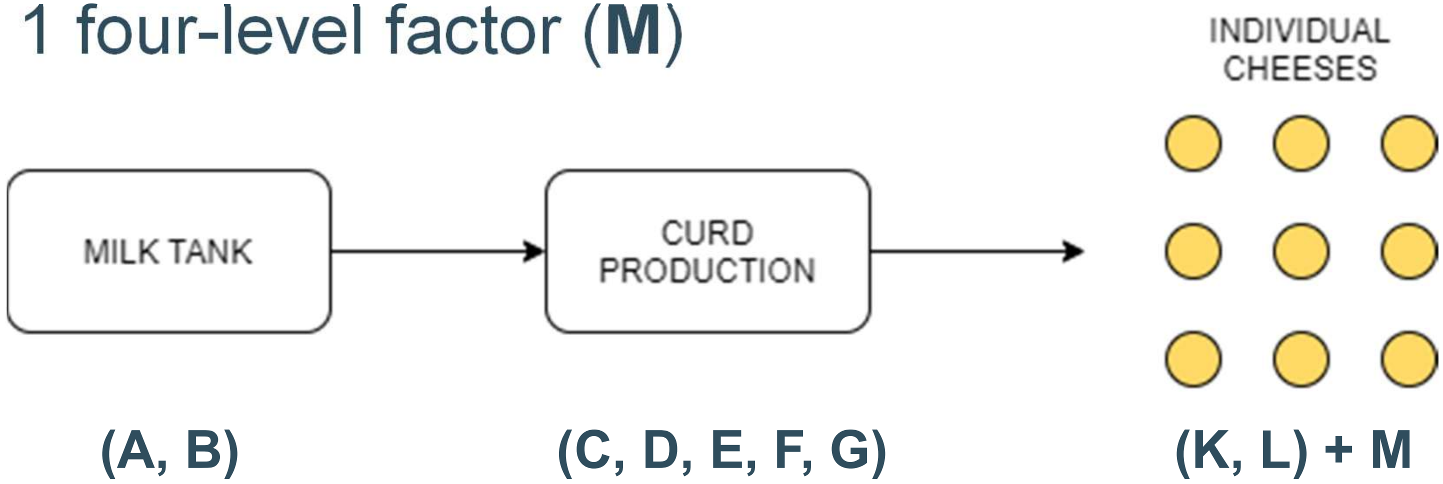
Four-level factors:

- Accomodate **categorical factors**
- Higher order effects in quantitative factors

Motivating example

Cheese making experiment (Schoen 1999):

- Screening experiment
- 128 runs
- 9 two-level factors (**A, B, C, D, E, F, G, K, L**)
- 1 four-level factor (**M**)



$4^1 2^9$ design in 128 runs

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Existing work

Few catalogs for four-and-two-level designs:

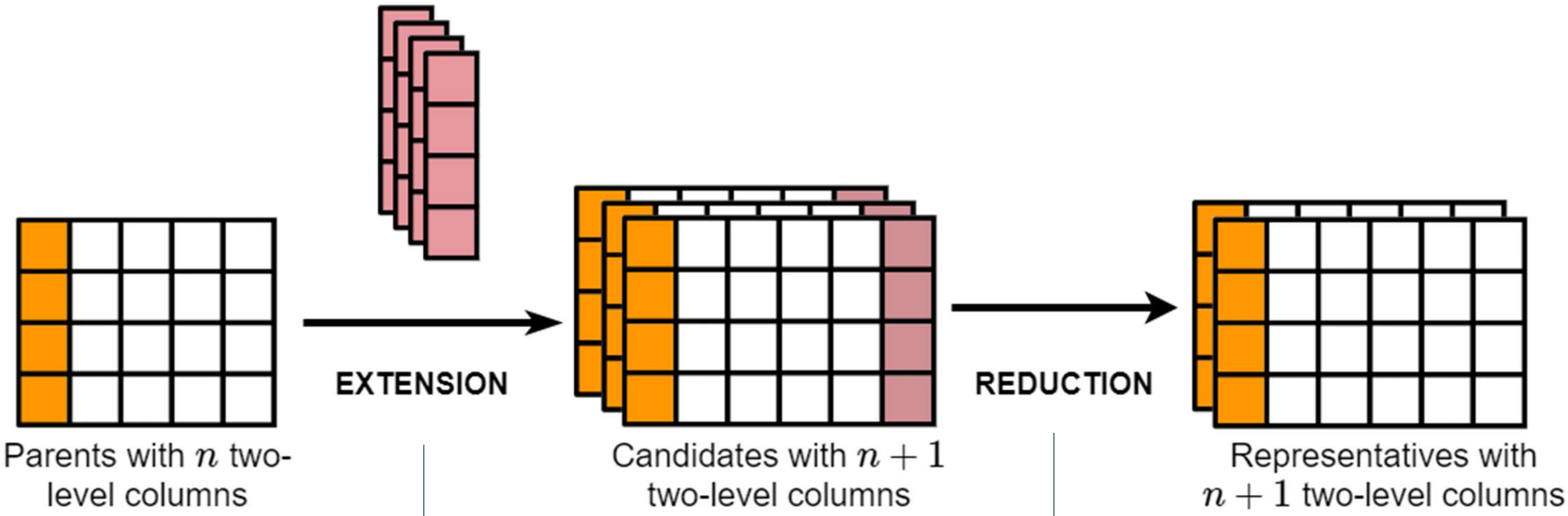
Author	Run sizes	Nbr four-level fac.	Nbr two-level fac.
Wu and Zhang (1993)	16, 32	1, 2	≤ 11
Ankeman (1999)	16, 32	1, 2, 3	≤ 14

Problems:

- Does not help for our example (128 runs)
- Both catalogs only showed minimum aberration designs

No complete catalog for large run sizes

Global enumeration procedure



Adding a two-level factor to the design

Selecting only non-equivalent designs

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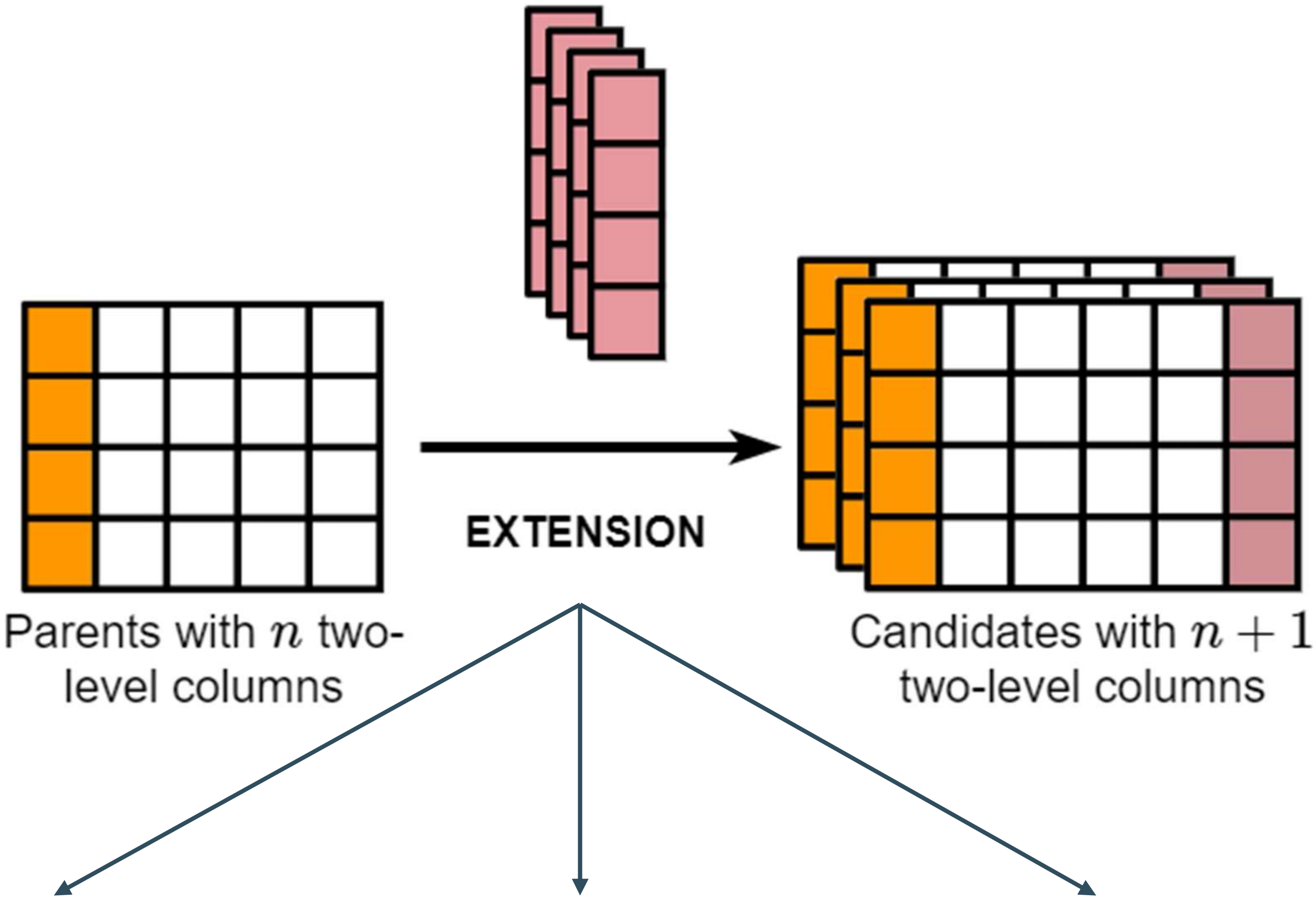
Extension methods

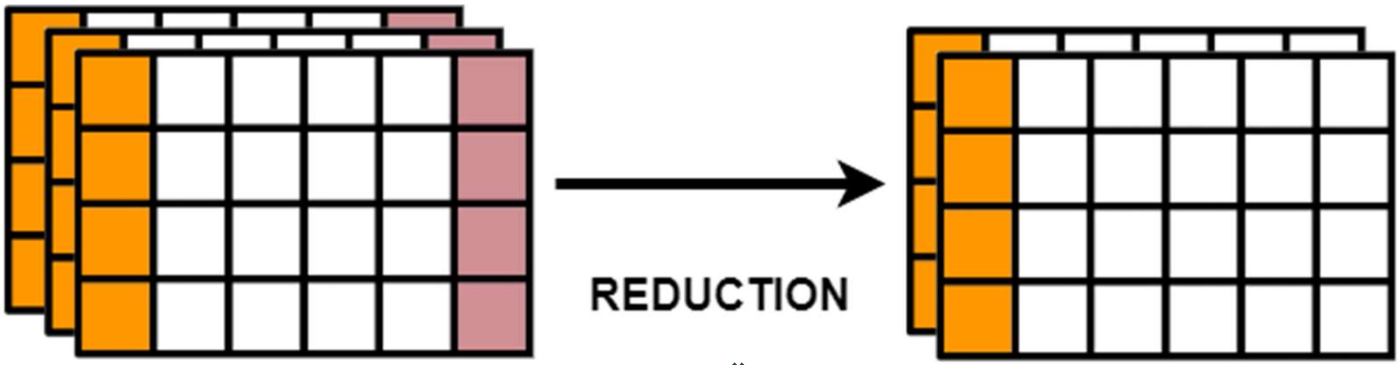


Search-table
Bingham and Sitter (1999)

DOP
Xu (2009)

MCS algorithm
Schoen et al. (2010)

Reduction methods



Candidates with $n+1$ two-level columns

Representatives with $n+1$ two-level columns

NAUTY
Mc Kay and Piperno (2014)

LMC test
Schoen et al. (2010)

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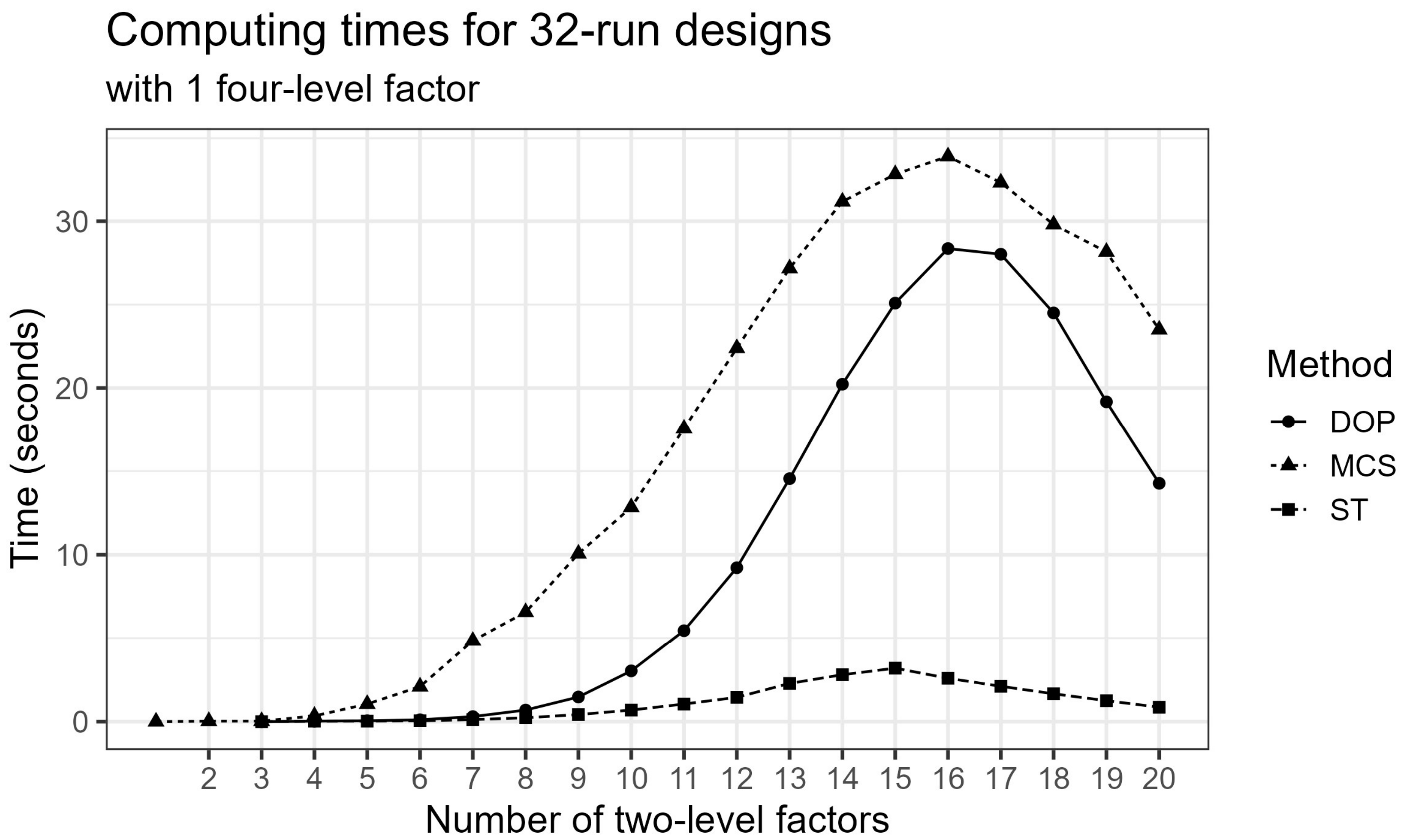
Selected procedures

- 3 Extension procedures + 2 Reduction procedures
- 6 combinations → 3 possible choices:

	ST	DOP	MCS
NAUTY	ST-NAUTY	DOP-NAUTY	Not optimal
LMC test	Not optimal	Incompatibe	MCS - LMC

Which one is the most efficient ?

Efficiency comparison



Repeated for 2 other test cases: same result for all

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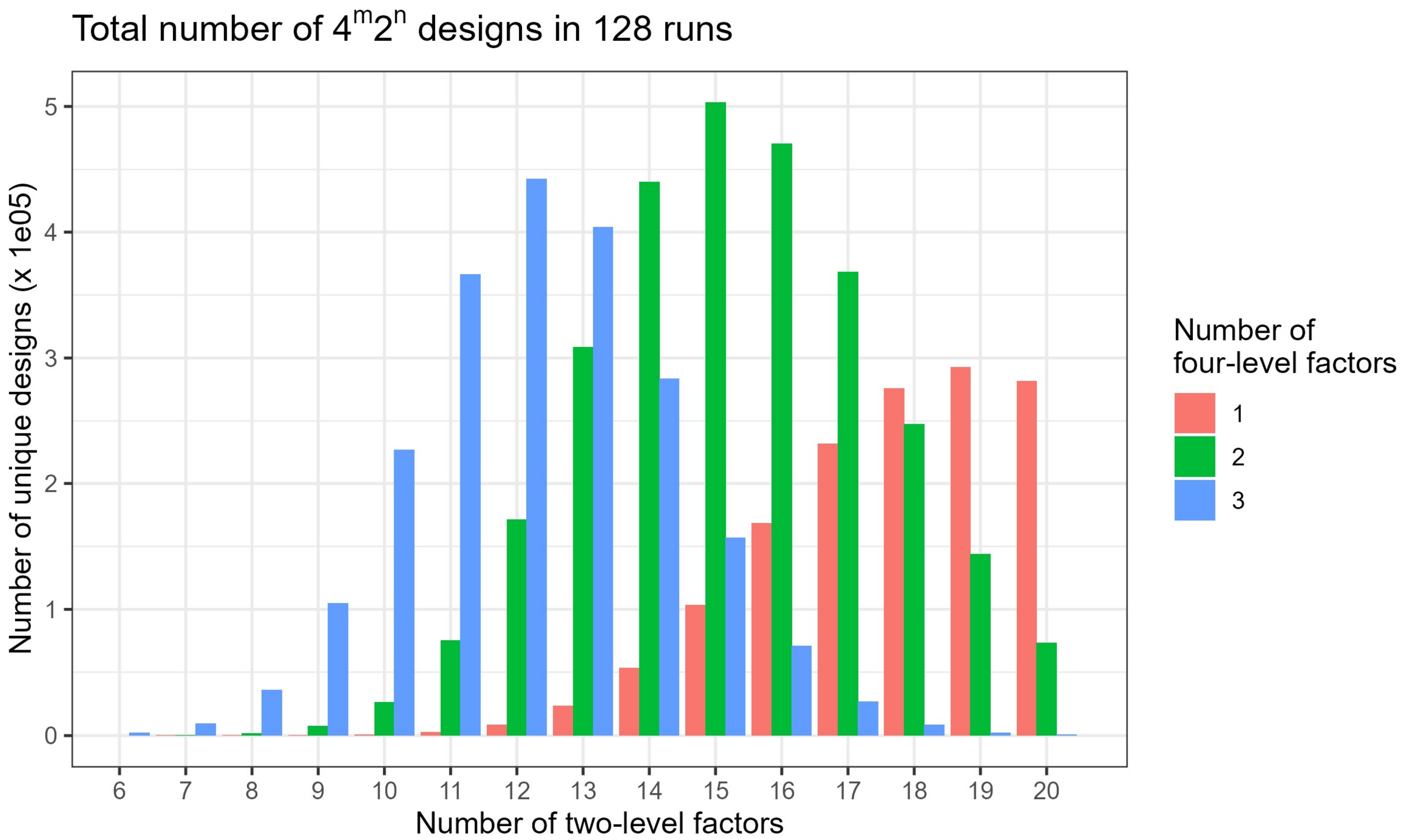
Catalog

- Enumeration using ST-NAUTY for:
  - **Two-level factors**: up to 20
  - **Run sizes**: 32, 64, 128 and 256
  - **Four-level factors**: 1, 2 or 3

- More than **6.5 million** designs enumerated *for now*
- Many 128-run and 256-run designs

128-run designs



Cheese-making experiment

- We are looking for : $4^1 2^9$ **128-run designs**
- From the catalog: **263 unique designs**
- Schoen (1999) could not use minimum aberration
- Catalog allows choice about:
  - Aberration
  - Interactions between 2FI
  - Any criterion ! → strength of the catalog

Conclusion

- **Enumeration**: ST-NAUTY is more efficient than DOP-NAUTY for four-and-two-level designs
- **Contribution**:
  - Efficient enumeration technique
  - Whole catalog for larger run sizes
- **Future work**:
  - Designs characterization
  - Pareto optimality selection